by one magnitude, that one magnitude to be always interpreted as a number. This radical innovation, the creation of this epoch-marking paradox, is due to Newton. Newton takes this vast step explicitly and consciously. The lectures which he delivered as Lucasian professor at Cambridge were published under the title, 'Arithmetica Universalis.' At the beginning of his Arithmetica Universalis he says, page 2: "Per Numerum non tam multitudinem unitatum quam abstractam quantitatis cujusvis ad aliam ejusdem generis quantitatem quae pro unitate habetur rationem intelligimus." [In quoting this, Pringsheim, p. 51, misses the first word. He omits the *Per*.]

As Wolf puts it (1710): "Number is that which is to unity as a piece of a straight line [a sect] is to a certain other sect." Thus the length of any sect is a real number, and the length of any possible sect incommensurable with the unit sect is an irrational number.

Says Hayward in his Vector Algebra (1892), page 5: "Number is essentially *discrete* or *discontinuous*, proceeding from one value to the next by a finite increment or jump, and so cannot, except in the way of a limit, represent, relatively to a given unit, a continuous magnitude for which the passage from one value to another may always be conceived as a *growth* through every intermediate value."

But the moment we accept Newton's definition of number it takes on whatever continuity is possessed by the sect. However, from this alone does not follow that for every irrational there is a sect whose length would give that irrational. G. Cantor was the first to bring out sharply that this is neither self-evident nor demonstrable, but involves an essential pure geometric assumption.

To free the foundations of general arithmetic from such *geometric* assumption, G. Cantor and Dedekind each developed his pure arithmetic theory of the irrational.

Professor Fine, in his 'Number-System of Algebra,' seems to miss this point completely. He gives, page 42, what purports to be a demonstration that "corresponding to every real number is a point on the line, the distance of which from the null-point is represented by the number," without any mention of the geometric assumption necessary, and then proceeds, page 43, to borrow the continuity of his number system from the naïvely supposed continuity of the line, the very thing for the avoidance of which G. Cantor and Dedekink made their systems.

Says Dedekind. "Um so schöner erscheint es mir, dass der Mensch ohne jede Vorstellung von messbaren Grössen, und zwar durch ein endliches System einfacher Denkschritte sich sur Schöpfung des reinen, stetigen Zahlenreiches aufschwingen kann; und erst mit diesem Hülfsmittel wird es ihm nach meiner Ansicht möglich, die Vorstellung von stetigen Raume zu einer deutlichen auszubilden."

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*The Tides and Kindred Phenomena in the Solar System.* By GEORGE HOWARD DARWIN. Boston and New York, Houghton, Mifflin & Co. 1898. Pp. xviii + 378.

During October and November, 1897, Professor Darwin delivered a course of semi-popular lectures on tidal phenomena at the Lowell Institute, Boston, Massachusetts. Since then the author has prepared these lectures for the press, and they are now, through the enterprise of Messrs. Houghton, Mifflin & Co., placed before the reading public in attractive book form.

The salient features of oceanic tides are more or less familiar to most people in these days. Indeed, some intelligent people will tell us that it is only necessary to read the daily papers of the seaboard towns, or to look in 'The Farmers' Almanac,' to learn when high and low water will occur. The educated public was not always so well informed, however. When, for example, Alexander the Great attempted to make a landing at the mouth of the Indus his fleet was nearly overwhelmed by the inrush of the tide. "The nature of the ocean," according to his biographer, Curtius, "was unknown to the multitude, and grave portents and evidences of the wrath of the gods were seen in what happened." The admirals of the present day know more about tides than the admiral of Alexander, and the wrath of a court of enquiry, rather than the wrath of the gods, hangs over the head of any commander who exposes his fleet to tidal dangers. But whence comes

the knowledge that enables us to anticipate the rise and fall of the ocean? How are the tidal tables of the daily papers and of 'The Farmers' Almanac' constructed?

It was the primary object of the lectures of Professor Darwin to answer such questions; to explain in a popular way, without the aid of mathematical and physical technicalities, how, from observations of the tides and from the modern theory thereof, predictions of the rise and fall of the ocean at any port may be issued years in advance. In addition to these more obvious tidal phenomena he has also discussed the more recondite phenomena of bodily tides in the earth and other members of the solar system. Thus, from questions of commercial or otherwise purely practical significance, the reader is led up to questions in cosmology of the highest scientific importance, especially in their bearings on the remote history, past and future, of our planet.

The task which Professor Darwin set for himself was a difficult one. Few, if any, questions in the mathematico-physical sciences are more profoundly complicated than those presented by tidal phenomena. Their elucidation has taxed the ingenuity of the most laborious investigators from the time of Newton to the present day. In the highly condensed language of mechanics it may be said that these phenomena, in any case, are simply the outcome of the energy, the angular momentum and the friction involved. But to turn conclusions expressed in such language into common parlance would seem to be almost as great a work as that of reaching the conclusions themselves. No one less well equipped than Professor Darwin would have dared to undertake this task. Thoroughly familiar with the details of tidal action, and himself a principal contributor to recent advances in tidal theories, he has produced a charmingly interesting and instructive book, which may be read with profit by those who know much as well as by those who know little of the tides.

The book is divided into twenty chapters under the following titles: Tides and Methods of Observation, Seiches in Lakes, Tides in Rivers—Tide Mills, Historical Sketch, Tide-generating Force, Deflection of the Vertical, The Elastic Distortion of the Earth's Surface by varying Loads, Equilibrium Theory of Tides, Dynamical Theory of the Tide Wave, Tides in Lakes—Cotidal Chart, Harmonic Analysis of the Tide, Reduction of Tidal Observations, Tide Tables, The Degree of Accuracy of Tidal Prediction, Chandler's Nutation—The Rigidity of the Earth, Tidal Friction, Tidal Friction (continued), The Figures of Equilibrium of a Rotating Mass of Liquid, The Evolution of Celestial Systems, Saturn's Rings. Each chapter is followed by a list of authorities on the subject of the chapter, and a good index completes the volume.

R. S. W.

*The Elements of Physics.* By ALFRED PAYSON GAGE. Boston, Ginn & Co. 1898. 12mo. Pp. x+381.

The author of this book put forth his first edition sixteen years ago and has long been favorably known as a reliable authority in the school room. The motto then adopted, 'Read Nature in the Language of Experiment,' is very properly retained in the present volume, which is not a revision, but a new book differing quite radically from the first in its method of presentation. The change, moreover, is a great improvement. We all agree that the experimental method is the proper method of investigating what is collectively called Nature, but there has been much difference of opinion about the advisability of regarding elementary pupils in the high school as fit to acquire their fundamental conceptions of physics by independent discovery. In the preface to the present volume Dr. Gage repeats the expression of his belief in the importance of the laboratory method in the high school, but adds that he has 'observed the development of a tendency which threatens seriously to impair its usefulness.' He is now 'convinced that both mental discipline and the acquisition of knowledge will be promoted if theory and experiment be somewhat sharply divided.'

There are a good many of us who have long held this last view in opposition to that which was carried out in Dr. Gage's first book. The demand for laboratory methods in the school room is much more than sixteen years old. A protest against the abuse of them was distinctly